Calculus Name_____ ID: 1

Study Guide

Date_____ Period____

For each problem, find the average rate of change of the function over the given interval.

1) $y = -x^2 + 2x - 1;\ \ [1, 2]$

2) $y = x^2 + 1;\ \ [1, 2]$

For each problem, find the instantaneous rate of change of the function at the given value.

3) $f(x) = \dfrac{1}{x + 1};\ \ 1$

4) $y = 2x^2 + x + 1;\ \ 0$

5) $y = -\dfrac{1}{x - 2};\ \ -3$

6) $f(x) = \dfrac{1}{x};\ \ 2$

Evaluate each limit.

7) $\lim\limits_{h \to 0} \dfrac{\left(\frac{5}{3} + h\right)^2 - \left(\frac{5}{3}\right)^2}{h}$

8) $\lim\limits_{h \to 0} \dfrac{\left(-\frac{1}{2} + h\right)^2 - \left(-\frac{1}{2}\right)^2}{h}$

9) $\lim\limits_{h \to 0} \dfrac{\tan\left(\frac{3\pi}{4} + h\right) - \tan\frac{3\pi}{4}}{h}$

10) $\lim\limits_{h \to 0} \dfrac{\sin\left(\frac{\pi}{3} + h\right) - \sin\frac{\pi}{3}}{h}$

11) $\lim_{h \to 0} \frac{\sqrt[3]{2+h} - \sqrt[3]{2}}{h}$

12) $\lim_{h \to 0} \frac{\sqrt{5+h} - \sqrt{5}}{h}$

Differentiate each function with respect to *x*.

13) $y = x^{\frac{4}{3}}$

14) $y = x^4$

15) $y = 4x^4$

16) $y = \sqrt[3]{x^2}$

17) $y = -3\sqrt[5]{x^2}$

18) $y = 5x^{\frac{1}{3}}$

19) $y = 3$

20) $y = -5$

21) $y = -\sqrt[4]{x}$

22) $y = 4\sqrt[5]{x}$

23) $y = 5\sqrt[3]{x}$

24) $y = 4x^{-2}$

25) $y = (5x^4 + 4)(x^3 + 5)$

26) $y = (5x^4 - 5)(4x^3 - 3)$

27) $y = (4x^4 - 4x^2 + 3)(x^5 - 3)$

28) $y = (-2x^5 + 3x^2 + 4)(4x^5 + 5)$

29) $y = \dfrac{x^2 + 2}{x^5 + 2}$

30) $y = \dfrac{4x^5 - x^2}{3x^5 + 2}$

31) $y = \dfrac{x^3 + 4x^2 + 4}{x^5 - 5}$

32) $y = \dfrac{5x^5 + 3x^2 + 3}{2x^5 + 2}$

For each problem, find the equation of the line tangent to the function at the given point. Your answer should be in slope-intercept form.

33) $y = 2x^2 - 4$ at $(0, -4)$

34) $y = -x^3 - 3x^2 + 5$ at $(1, 1)$

35) $y = \dfrac{3}{x - 3}$ at $(0, -1)$

36) $y = \dfrac{x^2}{4x - 4}$ at $\left(-2, -\dfrac{1}{3}\right)$

For each problem, find the equation of the line normal to the function at the given point. If the normal line is a vertical line, indicate so. Otherwise, your answer should be in slope-intecept form.

37) $y = x^3 - 7x^2 + 16x - 15$ at $(2, -3)$

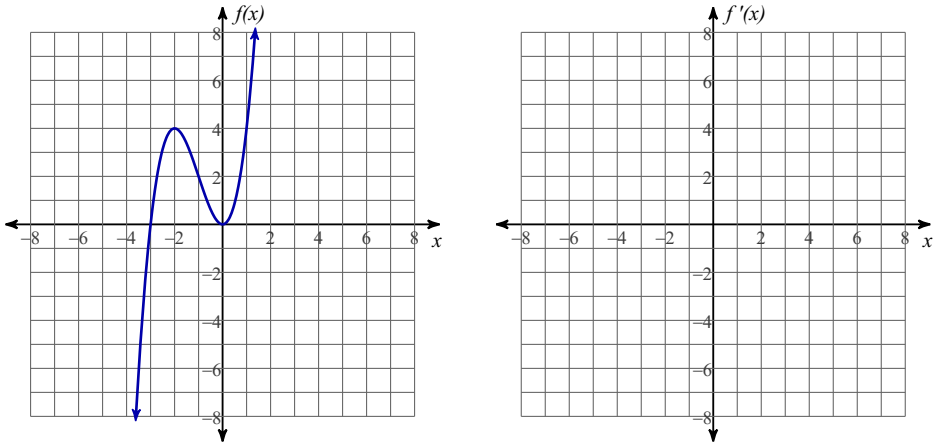
38) $y = -x^3 - 4x^2 - 4x - 4$ at $(-1, -3)$

39) $y = -\dfrac{x^2}{2} - 2x - 3$ at $(-2, -1)$

40) $y = -x^3 + x^2 - 3$ at $(-1, -1)$

Given the graph of $f(x)$, sketch an approximate graph of $f'(x)$.

41)



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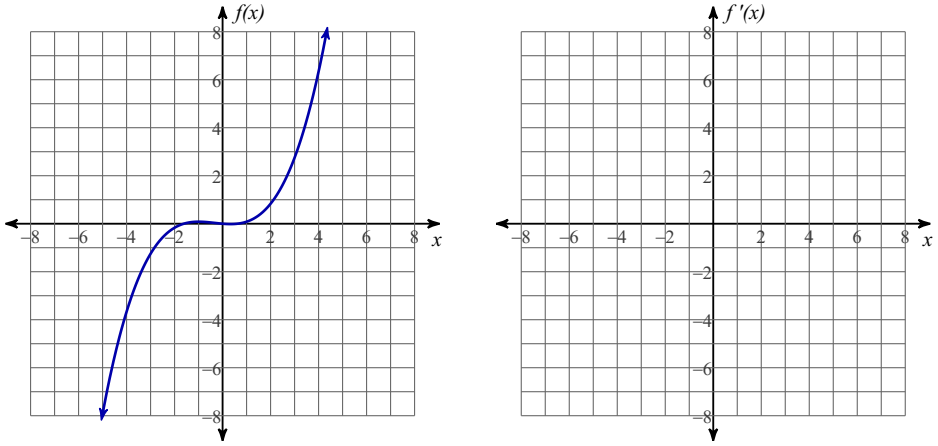


A particle moves along a horizontal line. Its position function is $s(t)$ for $t \geq 0$. For each problem, find the velocity function $v(t)$ and the acceleration function $a(t)$.

43) $s(t) = -t^3 + 30t^2 - 225t$

44) $s(t) = t^3 - 24t^2 + 144t$

Answers to Study Guide (ID: 1)

1) -1

2) 3

3) $-\frac{1}{4}$

4) 1

5) $\frac{1}{25}$

6) $-\frac{1}{4}$

7) $\frac{10}{3}$

8) -1

9) 2

10) $\frac{1}{2}$

11) $\frac{\sqrt[3]{2}}{6}$

12) $\frac{\sqrt{5}}{10}$

13) $\frac{dy}{dx} = \frac{4}{3}x^{\frac{1}{3}}$

14) $\frac{dy}{dx} = 4x^3$

15) $\frac{dy}{dx} = 16x^3$

16) $\frac{dy}{dx} = \frac{2}{3}x^{-\frac{1}{3}}$

17) $\frac{dy}{dx} = -\frac{6}{5}x^{-\frac{3}{5}}$

18) $\frac{dy}{dx} = \frac{5}{3}x^{-\frac{2}{3}}$

19) $\frac{dy}{dx} = 0$

20) $\frac{dy}{dx} = 0$

21) $\frac{dy}{dx} = -\frac{1}{4}x^{-\frac{3}{4}}$

22) $\frac{dy}{dx} = \frac{4}{5}x^{-\frac{4}{5}}$

23) $\frac{dy}{dx} = \frac{5}{3}x^{-\frac{2}{3}}$

24) $\frac{dy}{dx} = -8x^{-3}$

25) $\frac{dy}{dx} = (5x^4 + 4) \cdot 3x^2 + (x^3 + 5) \cdot 20x^3$
$= 35x^6 + 100x^3 + 12x^2$

26) $\frac{dy}{dx} = (5x^4 - 5) \cdot 12x^2 + (4x^3 - 3) \cdot 20x^3$
$= 140x^6 - 60x^3 - 60x^2$

27) $\frac{dy}{dx} = (4x^4 - 4x^2 + 3) \cdot 5x^4 + (x^5 - 3)(16x^3 - 8x)$
$= 36x^8 - 28x^6 + 15x^4 - 48x^3 + 24x$

28) $\frac{dy}{dx} = (-2x^5 + 3x^2 + 4) \cdot 20x^4 + (4x^5 + 5)(-10x^4 + 6x)$
$= -80x^9 + 84x^6 + 30x^4 + 30x$

29) $\frac{dy}{dx} = \frac{(x^5 + 2) \cdot 2x - (x^2 + 2) \cdot 5x^4}{(x^5 + 2)^2}$
$= \frac{-3x^6 - 10x^4 + 4x}{x^{10} + 4x^5 + 4}$

30) $\frac{dy}{dx} = \frac{(3x^5 + 2)(20x^4 - 2x) - (4x^5 - x^2) \cdot 15x^4}{(3x^5 + 2)^2}$
$= \frac{9x^6 + 40x^4 - 4x}{9x^{10} + 12x^5 + 4}$

31) $\frac{dy}{dx} = \frac{(x^5 - 5)(3x^2 + 8x) - (x^3 + 4x^2 + 4) \cdot 5x^4}{(x^5 - 5)^2}$
$= \frac{-2x^7 - 12x^6 - 20x^4 - 15x^2 - 40x}{x^{10} - 10x^5 + 25}$

32) $\frac{dy}{dx} = \frac{(2x^5 + 2)(25x^4 + 6x) - (5x^5 + 3x^2 + 3) \cdot 10x^4}{(2x^5 + 2)^2}$
$= \frac{-9x^6 + 10x^4 + 6x}{2x^{10} + 4x^5 + 2}$

33) $y = -4$

34) $y = -9x + 10$

35) $y = -\frac{1}{3}x - 1$

36) $y = \frac{2}{9}x + \frac{1}{9}$

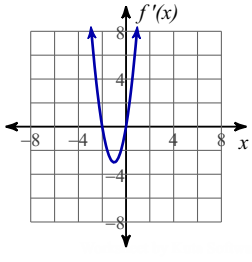
37) Normal line is vertical line at $x = 2$

38) $y = -x - 4$

39) Normal line is vertical line at $x = -2$

40) $y = \frac{1}{5}x - \frac{4}{5}$

41)



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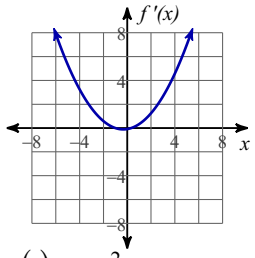

43) $v(t) = -3t^2 + 60t - 225$, $a(t) = -6t + 60$

44) $v(t) = 3t^2 - 48t + 144$, $a(t) = 6t - 48$