Name: key Date: ____________

AP CALCULUS BC
Unit 2 Review
Limits and Continuity

NO CALCULATOR IS ALLOWED ON THIS REVIEW.

1. What is $\lim_{x\to\infty} \frac{\sqrt{9x^2+2}}{4x+3}$? $= \lim_{x\to\infty} \frac{3x}{4x} = \frac{3}{4}$

(A) $\frac{3}{2}$ (B) $\frac{3}{4}$ (C) $\frac{\sqrt{2}}{3}$ (D) 1 (E) The limit does not exist.

2. What is $\lim_{x\to 1} \frac{\sqrt{x}-1}{x-1}$? $\left(\frac{\sqrt{x}+1}{\sqrt{x}+1}\right) \frac{x-1}{(x\sqrt{x}+x)-\sqrt{x}-1} = \frac{x-1}{x(\sqrt{x}+1)-1(\sqrt{x}+1)} = \frac{1}{\sqrt{x}+1}$

then substitute

$\lim_{x\to 1} \frac{1}{\sqrt{x}+1} = \frac{1}{\sqrt{1}+1} = \frac{1}{2}$

(A) 0

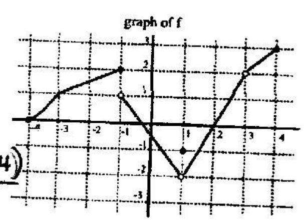
(B) $\frac{1}{2}$

(C) 1

(D) $\frac{3}{2}$

(E) The limit does not exist.

3. The function f is defined on the interval $[-5, 5]$ and its graph is shown to the right. Which of the following statements are true?



~~I.~~ $\lim_{x\to 1} f(x) = -1$

$f(x)=2x-4$ II. $\lim_{h\to 0} \frac{f(2+h)-f(2)}{h} = 2$ $\frac{2(2+h)-4-(2(2)-4)}{h}$

III. $\lim_{x\to -1^+} f(x) = f(-3)$ $\frac{4+2h-4-0}{h} = 2$

(A) I only (B) II only (C) I and II only (D) II and III only (E) I, II, III

4. The function f is continuous at $x = 1$.

$(x+3)(3x+1)$

If $f(x) = \begin{cases} \dfrac{\sqrt{x+3} - \sqrt{3x+1}}{x-1} & \text{for } x \neq 1 \\ k & \text{for } x = 1 \end{cases}$ then $k =$

(A) 0 (B) 1 (C) $\frac{1}{2}$ (D) $-\frac{1}{2}$ (E) none of the above

$$\frac{\sqrt{x+3} - \sqrt{3x+1}}{x-1}\left(\frac{\sqrt{x+3} + \sqrt{3x+1}}{\sqrt{x+3} + \sqrt{3x+1}}\right)$$

$$\frac{x+3-3x-1}{x\sqrt{x+3} + x\sqrt{3x+1} - \sqrt{x+3} - \sqrt{3x+1}}$$

$$= \frac{-2x+2}{(x-1)(\sqrt{x+3} + \sqrt{3x+1}} = \frac{-2(\cancel{x-1})}{\cancel{(x-1)}(\sqrt{x+3} + \sqrt{3x+1}} = \frac{-2}{\sqrt{1+3} + \sqrt{3(1)+1}} = \frac{-2}{2+2} = \boxed{-1/2}$$

5. Which of the following is true about the function f if $f(x) = \dfrac{(x-1)^2}{2x^2 - 5x + 3}$? $= \dfrac{(x-1)(x-1)}{(2x-3)(x-1)}$

I. f is continuous at $x = 1$.

II. The graph of f has a vertical asymptote at $x = 1$.

III. The graph of f has a horizontal asymptote at $y = \frac{1}{2}$.

(A) I only (B) II only (C) III only (D) II and III only (E) I, II, III

6. Which function is NOT continuous everywhere?

~~(A)~~ $y = |x|$

~~(B)~~ $y = x^{2/3}$

~~(C)~~ $y = \sqrt{x^2 + 1}$

~~(D)~~ $y = \dfrac{x}{x^2 + 1}$

(E) $y = \dfrac{4x}{(x+1)^2}$

7. $\lim_{x\to1} \frac{x^2+2x-3}{x^2-1} = \lim_{x\to1} \frac{(x+3)\cancel{(x-1)}}{(x+1)\cancel{(x-1)}}$

(A) −2

(B) −1

(C) 10

(D) 1

(E) 2

$\lim_{x\to1} \frac{(x+3)}{(x+1)}$

$\frac{1+3}{1+1} = \frac{4}{2}$

8. $\lim_{x\to2} \frac{\frac{1}{x-4}+\frac{1}{x}}{2-x} =$

$\lim_{x\to2} \frac{x+x-4}{x^2-4x} \cdot \frac{1}{2-x}$

$\lim_{x\to2} \frac{2\cancel{(x-2)}}{x(x-4)\cdot -1\cancel{(x-2)}}$

$= \frac{\cancel{2}}{\cancel{2}(2-4)} = \boxed{\frac{1}{-2}}$

9. $\lim_{x\to0} 4\frac{\sin x\cos x-\sin x}{x^2} = 4\lim_{x\to0} \frac{\sin x\,(\cos x-1)}{x^2}$

$\lim_{x\to0} \frac{\sin(ax)}{ax} = 1$

$\lim_{x\to0} \frac{1-\cos ax}{ax} = 0$

(A) 2 (B) $\frac{40}{3}$ (C) ∞ (D) 0 (E) undefi

$\lim_{x\to0} \frac{\sin x}{x} \cdot \frac{\cos x-1}{x}$

$= 1\cdot 0$

$= 0$

10. $\lim_{n \to \infty} \dfrac{3n^3 - 5n}{n^3 - 2n^2 + 1}$ is

(A) -5 (B) -2 (C) 1 (D) 3 (E) nonexistent

11. $\lim_{\theta \to 0} \dfrac{1 - \cos\theta}{2\sin^2\theta}$ is $\lim_{\theta \to 0} \dfrac{1-\cos\theta}{2(1-\cos^2\theta)} = \lim_{\theta \to 0} \dfrac{\cancel{1-\cos\theta}}{2(1+\cos\theta)\cancel{(1-\cos\theta)}} = \lim_{\theta \to 0} \dfrac{1}{2(1+\cos\theta)}$

(A) 0 (B) $\frac{1}{8}$ (C) $\frac{1}{4}$ (D) 1 (E) nonexistent

$= \dfrac{1}{2(1+1)}$

$= \dfrac{1}{4}$