Practice Quiz 5.1

1) $y = x^3 + 9x^2 + 24x + 20$ $[-5, -3]$

check endpts b/c closed interval

$y' = 3x^2 + 18x + 24$

$0 = x^2 + 6x + 8$

$0 = (x+2)(x+4)$

$x = -2, -4$ ← do not include -2 b/c not in interval

x	-5	-4	-3
f(x)	0	4	2

(-5, 0) absolute min
(-4, 4) absolute max

$(-5)^3 + 9(5)^2 + 24(-5) + 20$

$-125 + 225 - 120 + 20 = 0$

24, 4, 96

$(-4)^3 + 9(-4)^2 + 24(-4) + 20$

96, 64, 160

$-64 + 144 - 96 + 20$

$-160 + 164 = 4$

24, 3, 2

$(-3)^3 + 9(-3)^2 + 24(-3) + 20$

$-27 + 81 - 72 + 20$

27, 72, 99, 81, 18

$-18 + 20 = 2$

2) $y = x^3 + x^2 - x - 1$ $(-2, 0)$

$y' = 3x^2 + 2x - 1$

$0 = (3x - 1)(x + 1)$

$x = 1/3, -1$

x	-1	1/3
f(x)	0	-32/27

max (-1, 0)
min (1/3, -32/27)

$(-1)^3 + (-1)^2 - (-1) - 1$

$-1 + 1 + 1 - 1 = 0$

$(1/3)^3 + (1/3)^2 - (1/3) - 1$

$\frac{1}{27} + \frac{1}{9} - 1/3 - 1$

$\frac{1}{27} + \frac{3}{27} - \frac{9}{27} - \frac{27}{27} = \frac{-32}{27}$

3) $y = x^3 - 7x^2 + 16x - 12$

$y' = 3x^2 - 14x + 16$

$0 = (3x - 8)(x - 2)$

$x = 8/3, 2$

$(8/3, -4/27)$ min

$(2, 0)$ max

$y = (8/3)^3 - 7(8/3)^2 + 16(8/3) - 12$

$= \frac{512}{27} - \frac{448}{9} + \frac{128}{3} - 12$

$= -4/27$

$y = (2)^3 - 7(2)^2 + 16(2) - 12$

$= 8 - 28 + 32 - 12$

$= 0$

4) $y = -x^4 + 2x^2 + 4$

$y' = -4x^3 + 4x$

$0 = -4x(x^2 - 1)$

$x = 0, 1, -1$

$(-1, 5)$ max

$(0, 4)$ min

$(1, 5)$ max

5) Mean Value Theorem $f'(c) = \frac{f(b) - f(a)}{b - a}$

$y = -x^3 + 3x^2 + 2$ $[-1, 2]$

$y' = -3x^2 + 6x$

$(-1, 0)$

$(2, 6)$

$-3x^2 + 6x = \frac{6 - 0}{2 - -1}$

$-3x^2 + 6x = 0$

$-3x(x - 2) = 0$

$\boxed{x = 0, 2}$

6) $y = \frac{1}{2}x^2 + 3x + 7/2$ $[-7, -2]$

$y' = x + 3$

$(-7, 7)$

$(-2, -1/2)$

$x + 3 = \frac{7 - -1/2}{-7 - -2} = \frac{15/2}{-5}$

$= \frac{15}{2} \cdot \frac{1}{-5}$

$x + 3 = -\frac{3}{2}$

$\boxed{x = -9/2}$

Practice Quiz 5.1

7) $y = 2x^2 + 16x + 29 \quad [-6, -4]$

$(-6, 5)$
$(-4, -3)$

$y' = 4x + 16 = \frac{5 - -3}{-6 - -4}$

$4x + 16 = \frac{8}{-2}$

$4x + 16 = -4$

$-16 \quad -16$

$4x = -20$

$\boxed{x = -5}$

8) $y = -x^3 + 11x^2 - 39x + 49$

$(2, 7)$
$(6, -5)$

$y' = -3x^2 + 22x - 39 = \frac{7 - -5}{2 - 6} = \frac{12}{-4} = -3$

$-3x^2 + 22x - 39 = -3$

$-3x^2 + 22x - 36 = 0$

$x = \frac{-22 \pm \sqrt{22^2 - 4(-3)(-36)}}{2(-3)}$

$\boxed{x = \frac{-22 \pm \sqrt{52}}{-6}}$

9) $y = \frac{1}{8}x^4 - \frac{1}{4}x^2 + \frac{1}{8}$

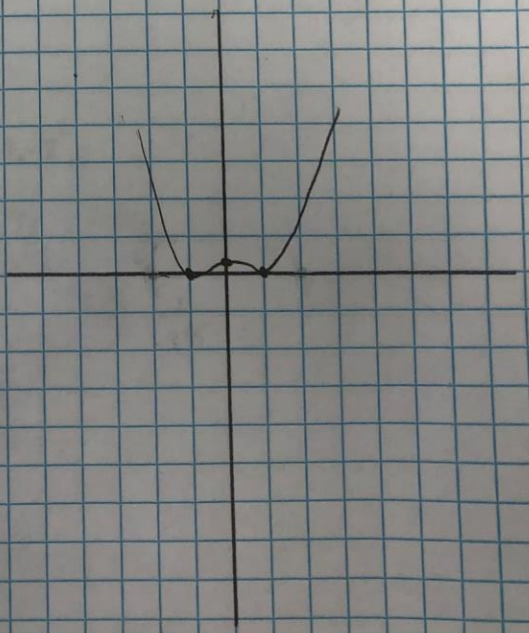
x-intercepts

y-intercept $(0, \frac{1}{8})$

$y' = \frac{1}{2}x^3 - \frac{1}{2}x$

$0 = x\left(\frac{1}{2}x^2 - 1\right)$

$x = 0, \pm\sqrt{2}$ ← critical pts.



$y' = \frac{3}{2}x^2 - \frac{1}{2}$

$0 = \frac{3}{2}x^2 - \frac{1}{2}$

$\left(\frac{2}{3}\right)\frac{1}{2} = \frac{3}{2}x^2\left(\frac{2}{3}\right)$

$\frac{1}{3} = x^2$

$\pm\sqrt{\frac{1}{3}} = x$ inflections