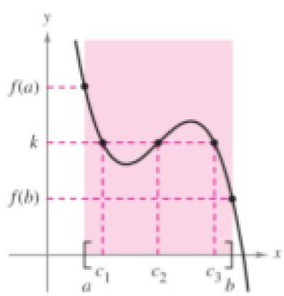
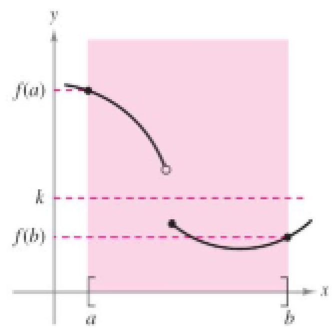
Intermediate Value Theorem

If f is continuous on the closed interval [a, b], $f(a) \neq f(b)$, and k is any number between f(a) and f(b), then there is at least one number c in the interval [a, b] such that $f(c) = k$.

Example: A person's height. If a person is 5 feet tall on their 13th birthday and 5'3" on their 14th birthday, then for any height h between 5' and 5'3", there must have been a time t when the height was exactly h.

The Intermediate Value Theorem guarantees the existence of at least one number c in the closed interval [a, b]. There may, of course, be more than one number c such that $f(c) = k$.

This only works for continuous functions!

| Example | Non Example |
| --- | --- |
|   *f* is continuous on [*a*, *b*].<br>[There exist three *c*'s such that $f(c) = k$.] |   *f* is not continuous on [*a*, *b*].<br>[There are no *c*'s such that $f(c) = k$.]<br>**Figure 1.36** |

Example:

Does the Intermediate Value Theorem guarantee a c-value on the given interval? $f(x) = x^3 + 3x^2 + 3x + 1, f(c) = 10, [0, 7]$

$$f(0) = (0)^3 + 3(0)^2 + 3(0) + 1 = 1$$

$$f(7) = (7)^3 + 3(7)^2 + 3(7) + 1 = 512$$

Yes. The Intermediate Value Theorem does guarantee a c-value because the function is both continuous and $f(c) = 0$ falls between $f(0) = 1$ *and* $f(7) = 512$.