6.4 Fundamental Theorem of Calculus

The Fundamental Theorem of Calculus, Part I: If f is continuous on [a, b], then the function

$$F(x) = \int_a^x f(t)dt$$

Has a derivative at every point x in [a, b] and

$$\frac{dF}{dx} = \frac{d}{dx}\int_a^x f(t)dt = f(x)$$

Super Important: $\frac{d}{dx}\int_a^x f(t)dt = f(x)$

Examples:

$\frac{d}{dx}\int_4^x t^2\,dt =$

$\frac{d}{dx}\int_\pi^x \ln t\,dt =$

$\frac{d}{dx}\int_0^x e^{4t^2-1}\,dt =$

$\frac{d}{dx}\int_x^5 \left(9t^3 - \cos t\right)dt =$

$\frac{d}{dx}\int_{-6}^{\sin x} \left(t^3 - 4t\right)dt =$

$\frac{d}{dx}\int_0^{x^3-3x} \frac{1}{1-t^2}\,dt =$

$\frac{d}{dx}\int_x^{x^2} \frac{1}{t}\,dt =$

$\frac{d}{dx}\int_{2x}^{x^2} (\tan t + 5)dt =$

AP Problem:

Let $g(x) = \frac{d}{dx}\int_0^x \sqrt{t^2+9}\,dt\,.\,What\ is\ g(-4)?$

a. -5   b. -3   c. 3   d. 4   e. 5

Fundamental Theorem of Calculus Part 2: If f is continuous at every point of [a, b], and if F is any antiderivative of f on [a, b], then

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

This part of the Fundamental Theorem is also called the Integral Evaluation Theorem.

Examples:

Evaluate:

$\int_{-1}^{3} \left(x^3 + 1\right)dx$

$\int_{0}^{5} e^x\,dx$

$\int_{1}^{4} \frac{1}{x}\,dx$

$\int_{0}^{\frac{\pi}{4}} (\cos x - 1)dx$