6.3 Definite Integrals and Antiderivatives

EXAMPLES

Suppose f and g are integrable functions such that $\int_{-1}^{1} f(x)\,dx = 5$, $\int_{1}^{4} f(x)\,dx = -2$, and $\int_{-1}^{1} g(x)dx = -4$. Find each of the following:

a) $\int_{4}^{1} f(x)dx$ b) $\int_{-1}^{4} f(x)dx$ c) $\int_{0}^{1} f(x)dx$

d) $\int_{-1}^{1} [2f(x)+3g(x)]dx$ e) $\int_{-1}^{4} [f(x)+g(x)]dx$ f) $\int_{-2}^{2} f(x)dx$

Average (Mean) Value: If f is integrable on [a, b], its average (mean) value on [a, b] is

$$av(f) = \frac{1}{b-a}\int_a^b f(x)dx$$

The Mean Value Theorem for Definite Integrals: If f is continuous on [a, b], then at some point c in [a, b],

$$f(c) = \frac{1}{b-a}\int_a^b f(x)\,dx$$

Fundamental Theorem of Calculus, Part I: If f is continuous on [a, b], then the function

$$F(x) = \int_a^x f(t)dt$$

Has a derivative at every point x in [a,b], and

$$\frac{dF}{dx} = \frac{d}{dx}\int_a^x f(t)dt \ = f(x).$$

Examples:

Together	On your own
$\frac{d}{dx}\int_4^x t^2 dt =$	$\frac{d}{dx}\int_{-1}^x \cos t\,dt =$
$\frac{d}{dx}\int_\pi^x \ln t\,dt =$	$\frac{d}{dx}\int_0^x \frac{1}{1+t^2}dt =$
$\frac{d}{dx}\int_1^{x^2} e^t dt =$	$\frac{d}{dx}\int_4^{2x} \frac{1}{1+t^2}dt =$
$\frac{d}{dx}\int_4^{3x} \cos t\,dt =$	$\frac{d}{dx}\int_{12}^{3x^2-x} \sin^2 x\,dt =$