6.2 Definite Integrals

Riemann Sums: Let f be a function defined on a closed interval [a, b]. For any partition P of [a, b], let the numbers c_k be chosen arbitrarily in the subintervals $[x_{k-1}, x_k]$. If there exists a number I such that

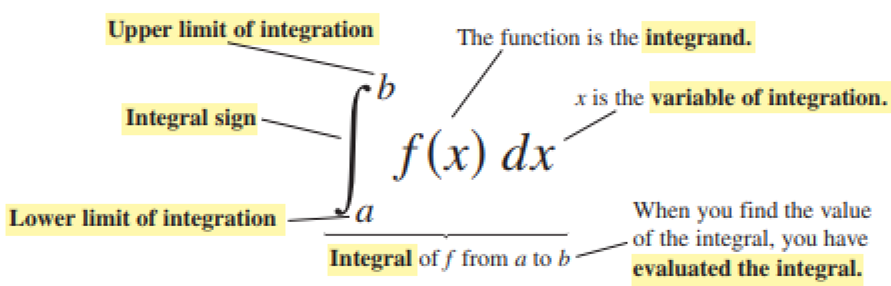
$$\lim_{\|P\| \to 0} \sum_{k=1}^{n} f(c_k)\,\Delta x = I$$

No matter how P and the c_k's are chosen, then f is integrable on [a, b] and I is the definite integral of f over [a, b].

**P is the partition and the $\|P\|$ denotes the norm of the partition and it tends toward zero. Basically, the rectangles are getting more and more narrow.

The Existence of Definite Integrals: All continuous functions are integrable. That is, if a function f is continuous on an interval [a, b], then its definite integral over [a, b] exists.

The Definite Integral of a Continuous Function on [a, b]: Let f be continuous on [a, b], and let [a, b] be partitioned into n subintervals of equal length $\Delta x = \frac{b-a}{n}$. Then the definite integral of f over [a, b] is given by $\lim_{n \to \infty} \sum_{k=1}^{n} f(c_k)\Delta x$, where each c_k is chosen arbitrarily in the k^{th} subinterval.



Area Under a Curve: If $y = f(x)$ is a nonnegative and integrable over a closed interval [a, b], then the area under the curve $y = f(x)$ *from a to b* is the integral of f from a to b.

$$A = \int_a^b f(x)dx$$

Area below the x-axis is considered negative thus:

$$A = -\int_a^b f(x)dx$$

Using fnInt: Example: $\int_0^3 (-160)dx$=480

Properties and Rules of Definite Integrals:

$\frac{d}{dx}[C] = 0$	$\int 0dx = C$
$\frac{d}{dx}[kx] = k$	$\int kdx = kx + C$
$\frac{d}{dx}[kf(x)] = kf'(x)$	$\int kf(x)dx = k\int f(x)dx$
$\frac{d}{dx}[f(x) \pm g] = f'(x) \pm g'(x)$	$\int [f(x) \pm g(x)dx = \int f(x)dx \pm \int g(x)\,dx$
$\frac{d}{dx}[x^n] = nx^{n-1}$	$\int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$
$\frac{d}{dx}[\sin x] = \cos x$	$\int \cos x\,dx = \sin x + C$
$\frac{d}{dx}[\cos x] = -\sin x$	$\int \sin x\,dx = -\cos x + C$
$\frac{d}{dx}[\tan x] = \sec^2 x$	$\int \sec^2 x\,dx = \tan x + C$
$\frac{d}{dx}[\sec x] = \sec x \tan x$	$\int \sec x\ \tan x = \sec x + C$
$\frac{d}{dx}[\cot x] = -\csc^2 x$	$\int \csc^2 x = -\cot x + C$
$\frac{d}{dx}[\csc x] = -\csc x \cot x$	$\int \csc x\ \cot x = -\csc x + C$
$\frac{d}{dx}[e^x] = e^x$	$\int e^x = e^x + C$
$\frac{d}{dx}[a^x] = (\ln a)\,a^x$	$\int a^x dx = \frac{1}{\ln a} * a^x + C$
$\frac{d}{dx}[\ln x] = \frac{1}{x}, x > 0$	$\int \frac{1}{x} dx = \ln\lvert x\rvert + C$