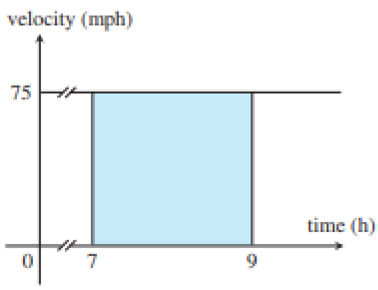
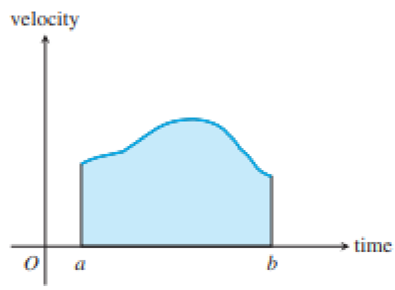
6.1 Estimating with Finite Sums

Consider the formula $distance = rate * time$. If you drive 80 mph for 3 hours, how far will you go?

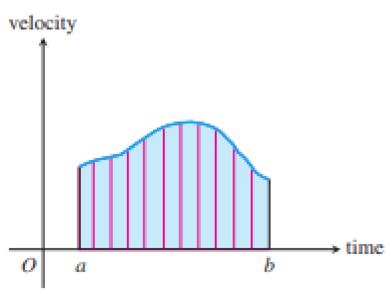
What is your speed/velocity was varied as a function of time. The graph would now look like.





The graph would no longer be a horizontal line, so the region under the graph is no longer a rectangle.

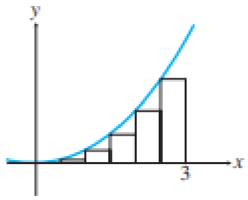
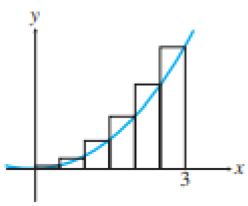
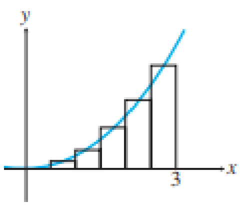
Will this area under the irregular region give the total distance? Imagine partitioning the region into many tiny subintervals, each one so small the velocity over it would essentially be constant.



The region is partitioned into vertical strips. If the strips are narrow enough, they are almost indistinguishable from rectangles. The sum of the area of these "rectangles" will give the total area and can be interpreted as distance traveled.

Rectangular Approximation Method (RAM):
We can approximate the area under the curve using Right-hand end points (RRAM), Left-hand end points (LRAM), or Midpoint end points (MRAM).

Example: $y = x^2$ from $[0, 3]$ with six subintervals.

LRAM:  	$(0)^2\left(\frac{1}{2}\right)+\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)+(1)^2\left(\frac{1}{2}\right)+\left(\frac{3}{2}\right)^2\left(\frac{1}{2}\right)+(2)^2\left(\frac{1}{2}\right)+\left(\frac{5}{2}\right)^2\left(\frac{1}{2}\right)=6.875$
RRAM:  	$\left(\frac{1}{4}\right)^2\left(\frac{1}{2}\right)+\left(\frac{3}{4}\right)^2\left(\frac{1}{2}\right)+\left(\frac{5}{4}\right)^2\left(\frac{1}{2}\right)+\left(\frac{7}{4}\right)^2\left(\frac{1}{2}\right)+\left(\frac{9}{4}\right)^2\left(\frac{1}{2}\right)+\left(\frac{11}{4}\right)^2\left(\frac{1}{2}\right)=8.9375$
MRAM:  	$\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)+(1)^2\left(\frac{1}{2}\right)+\left(\frac{3}{2}\right)^2\left(\frac{1}{2}\right)+(2)^2\left(\frac{1}{2}\right)+\left(\frac{5}{2}\right)^2\left(\frac{1}{2}\right)+(3)^2\left(\frac{1}{2}\right)=11.375$

Example: Using 4 subintervals find the LRAM, the RRAM and the MRAM of the following function, $f(x) = x^2 - x + 3, [0, 2]$

1. Graph the function.
2. Add appropriate rectangles.
3. Set up the sum. (Remember the width of the rectangle is the width of the division, the height is the function at that particular point.).