5.6 Linearization or Linear Approximation

Linearization: If you have a point and a derivative, you can use the tangent line to estimate another point on the function (a point that is very close to the point you already have.

**Concavity will influence if you estimate over or under.

Linearization: If f is differentiable at x = a, then the equation of the tangent line,

$$L(x) = f(a) + f'(a)(x - a),$$

Defines the linearization of f at a. The approximation $f(x) \approx L(x)$ is the standard linear approximation of f at a. The point x = a is the center of approximation.

Examples:

1. Use linearization to approximate the value of $f(1.2) if \frac{d}{dx}\left(f(x)\right) = -\frac{x}{y}$ and $f(x)$ passes through the point (2, 1).

1. Use the derivative and the ordered pair to find the slope of the tangent line at that particular point.	$m = \frac{d}{dx}\left(f(x)\right) = -\frac{x}{y} = -\frac{2}{1} = -2$
2. Use the slope of the tangent line and the point on the function to write the equation of the tangent line to the function at that point.	$y - 1 = -2(x - 2)$ $y = -2x + 5$
3. Then use the tangent line equation to approximate the value of f(1.2)	$f(1.2) = -2(1.2) + 5$ $f(1.2) = 2.6$

2. Find the linearization of $f(x) = \cos x \ at \ x = \frac{\pi}{2}$. Use it to estimate $f(\frac{9\pi}{16})$

1. Use the original function to find the complete ordered pair.	$f(x) = \cos x$ $f\left(\frac{\pi}{2}\right) = \cos\frac{\pi}{2} = 0$ $(\frac{\pi}{2}, 0)$
2. Find the derivative of the function and then the slope of the tangent line.	$\frac{d}{dx}(f(x) = \cos x)$ $f'(x) = -\sin x$ $m = f'\left(\frac{\pi}{2}\right) = -\sin\frac{\pi}{2} = -1$
3. Use the slope of the tangent line and the point on the function to write the equation of the tangent line to the function at that point.	$y - 0 = -1\left(x - \frac{\pi}{2}\right)$

	$y = -x + \frac{\pi}{2}$
4. Then use the tangent line equation to approximate the value of $f\left(\frac{9\pi}{16}\right)$	$f\left(\frac{9\pi}{16}\right) = -\frac{9\pi}{16} + \frac{8\pi}{16}$ *** Can check with graph.

AP Problems:

1. Let f be a differentiable function such that $f(3) = 2$ and $f'(3) = 5$. If the tangent line to the graph of f is used to find an approximation to a zero of f, that approximation is

a. 0.4	b. 0.5	**c. 2.6**	d. 3.4	e. 5.5

2.

x	-1.5	-1.0	-.05	0	0.5	1.0	1.5
$f(x)$	-1	-4	-6	-7	-6	-4	-1
$f'(x)$	-7	-5	-3	0	3	5	7

a. Write the equation of the tangent to the graph of f at the point where x = 1. Use this line to approximate the value of f(1.2).

b. Is the approximation greater or less than f(1.2)? How do you know?