5.5 Modeling and Optimization

Analyze and sketch the graph of $f(x) = x^4 - 12x^3 + 48x^2 - 64x$

Interval	$f(x)$	$f'(x)$	$f''(x)$	Characteristics
$-\infty < x < 1$				
$x = 1$				
$1 < x < 2$				
$x = 2$				
$2 < x < 4$				
$x = 4$				
$4 < x < \infty$				

Optimization: One of the most common applications of calculus involves determining the minimum or maximum values.

Finding max volume: A manufacturer wants to design an open box having a square base and a surface area of 108 square inches. What dimensions will produce a box with a max volume?

Because of the square base: $V = x^2 h$	This is called the primary equation because it gives the formula for the quantity to be optimized.
Surface Area $SA = area\ of\ base + area\ of\ 4\ sides$ $108 = x^2 + 4xh$	Because V is to be maximized, you want to write V as function of just one variable. Use substitution to solve for h.
$h = \frac{(108 - x^2)}{4x}$	
$V = \frac{x^2(108 - x^2)}{4x}$	Then use the h to find the volume.
To maximize V, take the derivative while considering the feasible domain $(0 \le x \le \sqrt{108})$	$\frac{dV}{dx} = 27 - \frac{3x^2}{4}$
Set equal to zero and solve for x. So V is maximum when x = 6.	$x = 6$

Guidelines for Solving Applied Minimum and Maximum Problems.

1. Identify all given quantities and all quantities to be determined. If possible sketch a picture.
2. Write a primary equation.
3. Reduce the primary equation to one having a single independent variable. (This may involve the use of secondary equations.)
4. Determine a feasible domain.
5. Determine the desired max or min by CALCULUS!

Examples:

1. You have been asked to design a one-liter oil can shaped like a right circular cylinder. What dimensions will use the least material?

2. Find two numbers whose sum is 20 and whose product is as large as possible.

3. A rectangle is to be inscribed under one arch of the sine curve. What is the largest area the rectangle can have, and what dimensions give that area?

4. What is the smallest perimeter possible for a rectangle whose area is $16in^2$, and what are its dimensions?