5.4 More Analyses of graphs using f' and f"

The first derivative of a function describes: ___________________________________

The second derivative of a function describes: _______________________________

Concave up: Means the graph is like a bowl.

Concave down: Means the graph is like a hill.

Point of Inflection: A point where the graph of a function has a tangent line and where the concavity changes is a **point of inflection**.

Example: Find all points of inflection of the graph of $y = e^{-x^2}$

a. Find the second derivative	$y = e^{-x^2}$ $y' = e^{-x^2} * (-2x)$ $y" = e^{-x^2}(-2x)(-2x) + e^{-x^2}(-2)$ $= e^{-x^2}(4x^2 - 2)$
b. It is tempting to oversimplify a point of inflection as a point where the second derivative is zero, but that can be wrong for 2 reasons. i. The second derivative can be a zero at a noninflection point. (Note that f' does not change sign at x = 0 of $f(x) = x^4$) ii. The second derivative need not be a zero at an inflection point. Example: $f(x) = \sqrt[3]{x}$ The concavity changes at x = 0 but there is a vertical tangent line, so both f'(0) and f"(0) fail to exist.	$0 = (e^{-x^2})(4x^2 - 2)$ $0 = (4x^2 - 2)$ $x = \pm\sqrt{\frac{1}{2}}$

Examples:

1. $f(x) = x^4 - 4x^3 + 2$

2. $f(x) = x^{\frac{2}{3}} - 3$

Second Derivative Test for Local Extrema:

- If $f'(c)$=0 and $f"(c) < 0$, then f has a local max at $x = c$.
- If $f'(c)$=0 and $f"(c) > 0$, then f has a local min at $x = c$

Use the Second Derivative Test to find the relative extrema of the following functions.

1. $f(x) = -\frac{1}{4}x^4 + \frac{9}{2}x^2 + 5$

2. $f(x) = xe^{-x}$

Connecting f, f', and f" numerically.

x	0	$0 < x < 1$	1	$1 < x < 2$	2	$2 < x < 3$	3	$3 < x < 4$
$f(x)$	-1	Negative	0	Positive	2	Positive	0	Negative
$f'(x)$	4	Positive	0	Positive	DNE	Negative	-3	Negative
$f"(x)$	-2	Negative	0	Positive	DNE	Negative	0	Negative

Let f be a function that is continuous on the interval [0,4). The function f is twice differentiable except at x =2. The function f and its derivatives have the properties indicated in the table above, where DNE indicates that the derivatives of f do not exist at x = 2.

a. For $0 < x < 4$, find all values of x at which f has relative extremum. Determine whether f has a relative maximum or relative minimum at each of these values. Justify your answer.
b. On the axes provided, sketch the graph of a function that has all the characteristics of f.