5.3 Analyses of Graphs using f and f'

Looking back:

1. A particle moves along the x-axis so that its position at any time $t \geq 0$ is given by $x(t) = t^3 - t^2 - t + 3$. For what values of t, $0 \leq t \leq 3$, is the particle's instantaneous velocity the same as its average velocity?

2. The table below shows the rate at which water is flowing out a pipe, in gallons per hour, at specific times. Is there a value of t, $0 \leq t \leq 24$ such that $R'(t) = 0$. Justify your answer.

t (hours)	0	3	6	9	12	15	18	21	24
R(t) Gallons/hr	9.6	10.4	10.8	11.2	11.4	11.3	10.7	10.2	9.6

Things to note: We should note that velocity v(t) is the antiderivative of the constant function 9.8. So we can write: $v(t) = 9.8t + C$. When a body moves from rest, v(0) = 0. We can solve and find C = 0; thus, $v(t) = 9.8t$. We also know that position is an antiderivative of v(t). So we can write it as $s(t) = 4.9t^2 + C$ Since s(0)=0 (starting position at starting time). We can write our position function as $s(t) = 4.9t^2$.

Example: 1. With what velocity will you hit the water if you step off from a 20-m diving platform? 2. With what velocity will you hit the water if you dive off the platform with an upward velocity of 2m/sec?

a. Start with the position function and solve for t.	$s(t) = 4.9t^2$ $20 = 4.9t^2$ $t = 2.02$
b. Use the found time in the velocity equation.	$v(t) = 9.8t$ $v(t) = 9.8(2.02)$ $v(t) = 19.8\ m/sec$
c. Use the velocity $v(t) = 9.8t + C$ but instead of v(0)=0, it now is v(0) = 2.	$v(t) = 9.8t + C$ $2 = 9.8(0) + C$ $C = 2$
d. For part 2, begin with $v(t) = 9.8t + C$. Because the initial velocity is 2 m/sec up, that is the opposite direction of gravity.	$v(t) = 9.8t + C$ $-2 = 9.8(0) + C$ $C = -2$
e. Then use the new v(t) to find the antiderivative/position function.	$v(t) = 9.8t - 2$ $s(t) = 4.9t^2 - 2t + C$ $-20 = 4.9(0)^2 - 2(0) + C$ $-20 = C$ $s(t) = 4.9t^2 - 2t - 20$
f. Use graphing calculator to find when t = 0. Then use that t in the v(t) to find the velocity.	$t = 2.235$ $v(2.235) = 9.8(2.235) - 2$ $= 19.903\ m/sec$

First Derivative Test for Local Extrema: The following test applies to continuous function f(x).

- If f’ changes from positive to negative at c, then there is a local **max** at c.
- If f’ changes from negative to positive at c, then there is a local **min** at c.
- If f’ does **not** change signs, then f has no local extreme value at c.

Concavity: The graph of a differentiable function y = f(x) is

- **Concave up** on an open interval I if y’ is increasing on I.
- **Concave down** on an open interval I if y’ is decreasing on I.

Concavity Test: the graph of a twice-differentiable function y = f(x) is

- **Concave up** on any interval where y”>0.
- **Concave down** on any interval where y”<0.