5.2 Mean Value Theorem, Rolle's Theorem, and Increasing and Decreasing Behavior

Find all the relative extrema for $y = 3x^4 - 16x^3 + 24x + 48$. Be sure to designate which are maxima and which are minima.

Use a calculator: Let f be the function with derivative given by $f'(x) = \sin(x^2 + 1)$. How many relative extrema does f have on the interval $2 < x < 4$?

Mean Value Theorem for Derivatives: If y = f(x) is continuous at every point of the closed interval [a, b] and differentiable at every point of its interior (a, b), then there is at least one point c in (a, b) at which

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

Examples:

1. Given $f(x) = x^2$ on the interval [0, 2]. Find a value for c such that $f'(c) = \frac{f(b)-f(a)}{b-a}$

$$f'(c) = \frac{2^2 - 0^2}{2 - 0} = \frac{4}{2} = 2$$

$$f'(c) = 2c = 2$$

$$c = 1$$

2. $f(x) = \sqrt{1 - x^2}, A = (-1, f(-1)), B = (1, f(1))$. Find a tangent in (-1, 1) that is parallel to $\overline{AB}$

1. Find the ordered pairs.	a. $(-1, 0) and\ (1, 0)$
2. Find the slope of $\overline{AB}$	b. $\frac{0-0}{1--1} = 0$
3. Find the derivative of f(x).	c. $f'(x) = \frac{-x}{\sqrt{1-x^2}}$
4. Set derivative equal to slope of $\overline{AB}$ and solve for x.	d. $0 = \frac{-x}{\sqrt{1-x^2}}$ e. $x = 0$ f. (0, 1) g. $y = 1$

3. Find the c guaranteed by the Mean Value Theorem for $y = x^3 - 2x^2 + 3$ on [-1, 2].

$$f'(c) = \frac{(2^3 - 2 * 2^2 + 3) - ((-1)^3 - 2(-1)^2 + 3)}{2 - -1} = \frac{3}{3} = 1$$

$$3x^2 - 4x = 1$$
$$3x^2 - 4x - 1 = 0$$

**Use the quadratic formula to solve for x.

Rolle's Theorem: Let f be a function defined on a closed [a, b]. If:

- f is continuous on [a, b]
- f is differentiable on (a, b), and
- $f(a) = f(b)$

Then there is at least one number c in the open interval (a, b) for which $f'(c) = 0$

Increasing/Decreasing Function Test: Let f be a function that is differentiable on the open interval (a, b):

- If $f'(c) > 0$ on (a, b), then f is increasing on (a, b).
- If $f'(c) < 0$ on (a, b), then f is decreasing on (a, b).

Example: Determine where the function $f(x) = 2x^3 - 9x^2 + 12x - 5$.

1. Find derivative.	a. $f'(x) = 6x^2 - 18x + 12$
2. Find critical points.	b. $0 = 6(x^2 - 3x + 2)$ $0 = (x-1)(x-2)$ $x = 1, 2$
3. Then set up a table with the intervals.	

Interval	X within Interval	$f(x) = 2x^3 - 9x^2 + 12x - 5$	$f'(x) = 6x^2 - 18x + 12$	Conclusion
$(-\infty, 1)$	0		12	Positive: Increasing
$(1, 2)$	1.5		-1.5	Negative: Decreasing
$(2, \infty)$	3		12	Negative: Increasing

Antiderivative: A function F(x) is an **antiderivative** of a function f(x) if F'(x) = f(x) for all x in the domain of f. This process is called **antidifferentiation.**

Example: $f'(x) = 2 \rightarrow\rightarrow\rightarrow f(x) = 2x + C$

*** C is a constant we don't know. $f(x) = 2x + 3$ *has the same derivative as* $f(x) = 2x - 1000$