4.2 notes: More Chain Rule

The Chain Rule and . . .

. . . the Product Rule: Use the <u>Product Rule and the Chain Rule</u> at the same time.

Example: Find the derivative of $f(x) = x^2\sqrt{1-x^2}$

- Rewrite: $f(x) = x^2(1-x^2)^{\frac{1}{2}}$
- Apply the Product Rule and Chain Rule:
  $f'(x) = 2x * (1-x^2)^{\frac{1}{2}} + \frac{1}{2}(-2x)(1-x^2)^{-\frac{1}{2}}(x^2)$
- Simplify: $f'(x) = 2x\sqrt{1-x^2} - \frac{x^3}{\sqrt{1-x^2}}$

. . .the Quotient Rule: Use the <u>Quotient Rule and the Chain Rule</u> at the same time.

Example: Find the derivative of $f(x) = \frac{x}{\sqrt[3]{x^2+4}}$

- Rewrite: $f(x) = \frac{x}{(x^2+4)^{\frac{1}{2}}}$
- Apply the Quotient Rule and Chain Rule:

$$f'(x) = \frac{(1)\sqrt[3]{x^2+4} - x * \frac{1}{3}(2x)(x^2+4)^{-2/3}}{(x^2+4)^{2/3}}$$

- Simplify: $f'(x) = \frac{(x^2+4)^{1/3} - \frac{2x^2}{3(x^2+4)^{2/3}}}{(x^2+4)^{2/3}}$
- Factor the numerator: $f'(x) = \frac{1}{3}(x^2+4)^{-2/3}[\frac{3(x^2+4)-(2x^2)(1)}{(x^2+4)^{2/3}}$
- Simplify: $f'(x) = \frac{x^2+12}{3(x^2+4)^{4/3}}$

Parametric functions and derivatives: A parametrized curve (x(t), y(t)) is differentiable at t if x and y are differentiable at t. At a point on a differentiable parametrized curve where y is also a differentiable function of x, the derivatives $dy/dt, dx/dt, and\ dy/dx$ are related by the Chain Rule.

$$\frac{dy}{dt} = \frac{dy}{dx} * \frac{dx}{dt} \quad or \quad \frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

If $\frac{dx}{dt} \neq 0$, we may divide both sides of this equation by $\frac{dx}{dt}$ to solve for $\frac{dy}{dx}$.

Example: Find the line tangent ot the right-hand hyperbola branch defined parametrically by $x = \sec t, y = \tan t,\ -\frac{\pi}{2} < t < \frac{\pi}{2}\ at\ the\ point\ (\sqrt{2}, 1)$, where $t = \pi/4$.

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$
$$= \frac{sec^2 t}{\sec t \tan t}$$
$$= \frac{\sec t}{\tan t}$$
$$= \csc t$$

Set $t = \frac{\pi}{4}$ gives $\csc\left(\frac{\pi}{4}\right) = \sqrt{2}$.

Then use this slope with the point to write the equation of the tangent line.

$$y - 1 = \sqrt{2}(x - \sqrt{2})$$
$$y = x\sqrt{2} - 2 + 1$$
$$y = x\sqrt{2} - 1$$