4.1 Chain Rule Notes

Composite Functions: Composite functions are the combination of two functions such as $f(x) and\ g(x)$.
One function replaces allof the x's in the other function.

Notation: $(f \circ g)(x)\ or\ f(g(x))$

Example: If $f(x) = x^2\ and\ g(x) = 2x + 1, then\ f(g(x)) = (2x + 1)^2$

Practice:

1. $f(x) = x^3,\ g(x) = sin\,x$ — Find: Find $(f \circ g)(x)$

2. $r(\theta) = 2\theta,\ s(\theta) = 3\theta + 5$ — Find: $(r \circ s)(\theta)$

3. $y(t) = \cos t,\ x(t) = t^2 - 1$ — Find: Find $x(y(t))$

Another Thought: You can think of two functions in the composite function as an "outside" and an "inside" function.
For example: $y = \cos^2 x$ can be rewritten as

$$y = (\cos x)^2, where\ the\ outside\ function\ is\ y = u^2 and\ inside\ as\ u = \cos x$$

Practice: Name the outside and inside functions of the following composite functions.

Function	Outside	Inside
1. $y = \sin(x^2 + 3)$		
2. $y = (3x - 2)^3$		
3. $y = \cos(x^2 + x)$		
4. $y = sec(\tan x)$		
5. $y = 4\sqrt{x^2 - 3}$		

Chain Rule: If f is differentiable at the point $u = g(x)$, and g is differentiable at x, then the composite function $(f \circ g)(x) = f(g(x))$ is differentiable at x, and its derivative

$$d/dx[f(g(x))] = f'(g(x)) \cdot g'(x)$$

In other words: If you have a composite function, the derivative of this function is the derivative of the outside function (leaving the inside function alone) multiplied by the derivative of the inside function.

Examples:
1. $y = \sin(x^2 + 3)$
2. $y = (3x - 2)^2$
3. $y = \cos(x^2 + x)$
4. $y = \sec(\tan x)$
5. $y = 4\sqrt{x^2 - 3x}$