3.7 Derivatives of Trigonometric Functions

Examples: Find the value of each of the following without a calculator.

a. $\cos(\sin^{-1}\frac{\sqrt{2}}{2}) = \frac{\sqrt{2}}{2}$ b. $\sec(\tan^{-1}\frac{1}{\sqrt{3}}) = \sec(\frac{\pi}{6}) = \frac{2}{\sqrt{3}}$ c. $\cos^{-1}(\cos\frac{5\pi}{4}) = \cos^{-1} -\frac{\sqrt{2}}{2} = \frac{5\pi}{4}$

Derivatives of Trig Functions

$$\frac{d}{dx}\sin x = \cos x \qquad \frac{d}{dx}\cos x = -\sin x$$

$$\frac{d}{dx}\tan x = sec^2 x \qquad \frac{d}{dx}\cot x = -csc^2 x$$

$$\frac{d}{dx}\sec x = \sec x \tan x \qquad \frac{d}{dx}\csc x = -\csc x \cot x$$

Examples: No Calculator. Find y' if

a. $y = x^2 \sin x$

Product Rule: $y' = 2x \sin x + x^2 \cos x$

b. $y = \frac{\cos x}{1-\sin x}$

Quotient Rule: $y' = \frac{-\sin x(1-\sin x)-\cos x(-\cos x)}{(1-\sin x)^2}$

Simplify: $y' = \frac{-\sin x+\sin^2 x+\cos^2 x}{(1-\sin x)^2} = \frac{1}{1-\sin x}$

c. Find y'' if $y = \sec x$

First Derivative: $y' = \sec x \tan x$

Product Rule: $y'' = \sec x \ \tan x \tan x + \sec x \sec^2 x$

Simplify: $y'' = \sec x \tan^2 x + \sec^3 x$

A weight hanging froma spring is stretched 5 units beyond its rest position (s=0) and released at time t = 0 to bob up and down. Its position at any time later t is given by the function $s = 5\cos t$. What are its velocity and acceleration at time t?

Calculator: Find the equations of the lines that are tangent and normal to the curve

$$f(x) = \frac{\tan x}{x} \ at \ x = 2$$