3.5 Velocity and Other Rates

Instantaneous Rate of Change: the instantaneous rate of change of f with respect to x at a is the derivative : $f'(a) = \lim_{h \to 0} \frac{f(a+h)-f(a)}{h}$, provided the limit exists.

Example 1:

a. Find the rate of change of the area A of a circle with respect to its radius r.
   i. $\frac{dA}{dr} = \frac{d}{dr}(\pi r^2) = \pi * 2r = 2\pi r$
b. Evaluate the rate of change of A at r =5 and r = 10.
   ii. $r = 5, then\ 2\pi(5) = 31.4$
   iii. $r = 10, then\ 2\pi(10) = 62.8$
c. If r is measured in inches and A is measured in square inches, what units would be appropriate for $dA/_{dr}$?
   iv. The appropriate units for $dA/_{dr}$ are square inches per inch.

Displacement: Displacement of an object over time interval from $t\ to\ \Delta t$ is $\Delta s = f(t + \Delta t) - f(t)$.

Average Velocity: $v_{av} = \frac{displacement}{travel\ time} = \frac{\Delta s}{\Delta t} = \frac{f(t+\Delta t)-f(t)}{\Delta t}$

Instantaneous Velocity: is the derivative of the position function $s = f(t)$ with respect to time. At time t the velocity is: $f(t) = \frac{ds}{dt} = \lim_{\Delta t \to 0} \frac{f(t+\Delta t)-f(t)}{\Delta t}$

Speed: Is the absolute value of velocity. Speed = $|v(t)| = \left|\frac{ds}{dt}\right|$

Acceleration: is the derivative of velocity with respect to time. If a body's velocity at time t is $v(t) = ds/_{dt}$, then the body's acceleration at time t is

$a(t) = \frac{dv}{dt} = \frac{d^2s}{dt^2}$

Free Fall Constants

- English units: $g = 32\frac{ft}{sec^2}$, $s = \frac{1}{2}(32)t^2 = 16t^2$ (s is in feet)
- Metric units: $g = 9.8\frac{m}{sec^2}$, $s = \frac{1}{2}(9.8)t^2 = 4.9t^2$ (s is in meters)

Example: A dynamite blast propels a heavy rock straight up with a launch velocity of 160 ft/sec. It reaches a height of $s = 160t - 16t^2$ after t seconds.

a. How high does the rock go?

i. The instant when the rock is at the highest point is the one instant during the flight when the velocity is 0. At any time t, the velocity is $v = \frac{ds}{dt} = \frac{d}{dt}(160t - 16t^2) = 160 - 32t$

$$160 - 32t = 0$$
$$t = 5\ sec.$$

Max height is $s(5) = 160(5) - 16(5)^2 = 400\ ft.$

b. What is the velocity and speed of the rock when it is 256 ft above the ground on the way up? On the way down?

ii. To find the velocity when the height is 256 ft, we determine the two values of t for which $s(t) = 256\ ft.$

$$s(t) = 160t - 16t^2 = 256$$
$$16t^2 - 160t + 256 = 0$$
$$16(t-2)(t-8) = 0$$
$$t = 2\ or\ 8\ seconds$$

Then substitute into the velocity equation:

$$v(2) = 160 - 32(2) = 96\ ft/sec$$
$$v(8) = 160 - 32(8) = \ -96\ ft/sec$$

c. What is the acceleration of the rock at any time t during its flight?

iii. $a = \frac{dv}{dt} = \frac{d}{dt}(160 - 32t) = \ -32\ ft/sec^2$

d. When does the rock hit the ground?

iv. The rock hits the ground at the positive time for which s = 0. The equation $160t - 16t^2 = 0$ has two solutions t = 0 and t = 10. The blast initiated the flight at t = 0. The rock returns at t = 10.