2.4 Rates of Change and Tangent Lines

Average Rate of Change: Slope. We can always think of an average rate of change as the slope of a secant line (the line connecting the two points on a curve).

Examples:

1. Determine the formula for the slope of the tangent to the curve $f(x)=x^2$. At the point (2, 4).
   a. Start with secant line through P(2, 4) and nearby point $Q(2\ +h,(2+h)^2)$
   b. Write an expression for slope:

   Secant Slope = $\frac{\Delta y}{\Delta x}=\frac{(2+h)^2-4}{h}$

   $$=\frac{h^2+4\,h+4-4}{h}$$
   $$=\frac{h^2+4\,h}{h}$$
   $$=h+4$$

   c. The limit of the secant slope as Q approaches P along the curve is

   $$\lim_{Q\to P}(secant\ slope)=\lim_{h\to 0}(h+4)=4$$

   d. This means the slope is 4. Then use the point P(2, 4) to find the equation of the line. $y=4x-4$.
2. For example 1, at what value of x will the slope of the tangent to the curve be -8?
3. Determine the formula for the slope of the tangent to the curve $f(x)=\frac{1}{x}$.
   a. Find the slope of the curve at x = a.

$$\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}=\lim_{h\to 0}\frac{\frac{1}{a+h}-\frac{1}{a}}{h}=\lim_{h\to 0}\frac{1}{h}*\frac{a-(a+h)}{a(a+h)}=\lim_{h\to 0}\frac{-h}{ha(a+h)}=\lim_{h\to 0}\frac{-1}{a(a+h)}=\frac{-1}{a^2}$$

   b. Where does the slope equal -1/4?

   $$\frac{-1}{a^2}=-\frac{1}{4}$$
   $$a^2=4$$
   $$a=\pm 2$$

   **Then plug $\pm 2\ into\ f(x)=\frac{1}{x}\ to\ find\ the\ ordered\ pairs\ \left(2,\frac{1}{2}\right)\ and\ \left(-2,-\frac{1}{2}\right)$

   c. What happens to the tangent to the curve at the point (a, 1/a) for different values of a? The slope $-1/a^2$ is always negative.
4. Find the formula for the slope of the tangent to the curve x^2-4x at $x=1$
   a. Write the equation for the tangent line at that point.
   b. Write the equation for the normal at that point.