2.2 Limits Involving Infinity

Horizontal Asymptote: The line $y = b$ is a horizontal asymptote of the graph of a function $y = f(x)$ if either $\lim_{x\to\infty} f(x) = b \ or \ \lim_{x\to-\infty} f(x) = b$.

Properties of Limits as $x \to \pm\infty$

If *L, M, c, and k* are real numbers and $\lim_{x\to\pm\infty} f(x) = L \ and \ \lim_{x\to\pm\infty} g(x) = M$	
1. Sum Rule $\lim_{x\to\pm\infty} f(x) + g(x) = L + M$ The limit of the sum of two functions is the sum of their limits.	2. Difference Rule $\lim_{x\to\pm\infty} f(x) - g(x) = L - M$ The limit of the difference of two functions is the difference of their limits.
3. Product Rule $\lim_{x\to\pm\infty} (f(x) * g(x)) = L * M$ The limit of a product of two functions is the product of their limits.	4. Constant Multiple Rule: $\lim_{x\to\pm\infty} \left(k * f(x)\right) = k * L$ The limit of a constant times a function is the constant times the limit of the function.
5. Quotient Rule: $\lim_{x\to\pm\infty} \frac{f(x)}{g(x)} = \frac{L}{M}, M \neq 0$ The limit of a quotient of two functions is the quotient of their limits, provided the limit of the denominator is not zero.	6. Power Rule: If *r and s* are integers, $s \neq 0$, then $\lim_{x\to\pm\infty} (f(x))^{r/s} = L^{r/s}$ Provided that $L^{r/s}$ is a real number. The limit of a rational power of a function is that power of the limit of the function, provided the latter is a real number.

Vertical Asymptote: A line $x = a$ is a vertical asymptote of the graph of a function $y = f(x)$ if either $\lim_{x\to a^+} f(x) = \pm\infty \ or \ \lim_{x\to a^-} f(x) = \pm\infty$

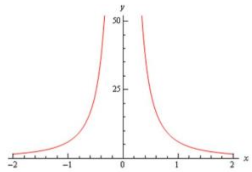
Finding limits: $\lim_{x\to c} f(x)$

1. Substitute
    a. Get a number—this is the limit
    b. $\frac{0}{0}$ try another strategy
        1. Factor and cancel
        2. Multiply by the conjugate

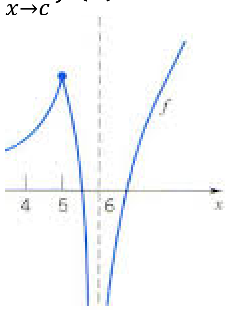
3. Use trig identities and limits
4. Simplify complex fractions
   **2 options—substitute again or move to c.

c. $\frac{n}{0}$—vertical asymptote

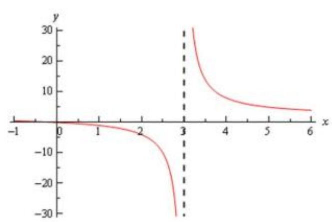
1. $\lim_{x \to c} f(x) = \infty$



2. $\lim_{x \to c} f(x) = -\infty$



3. $\lim_{x \to c} f(x) = D.N.E.$



*** Watch for negative exponents. Move first, then evaluate.