1.10 Functions and Logarithms

Log Function Definition: For $x > 0, and\ a \neq 1,$

$$y = \log_a x \ if\ and\ only\ if\ x = a^y$$

The function given by
$f(x) = \log_a x$ (read as log base a of x) is called the logarithmic function with base a.

Examples: Evaluate the following:

1. $\log_2 32 = 5$	2. $\log_3 \frac{1}{27} = -3$	3. $\log_7 1 = 0$

Properties of Logs

1. $\log_a 1 = 0\ because\ a^0 = 1$
2. $\log_a a = 1\ because\ a^1 = a$
3. $\log_a a^x = x\ and\ a^{\log_a x} = x$ (inverse properties)
4. If $\log_a x = \log_a y\ , then\ x = y$ (one-to one property)

The Natural Logarithmic Function:
For $x > 0$,

$y = \ln x$ if and only if $x = e^y$

The function given by $f(x) = \log_e x = \ln x$
Is called the natural log function.

Change of Base Formula: Let a, b, and x be positive real numbers such that $a \neq 1\ and\ b \neq 1$.
Then $\log_a x$ can be converted to a different base using the following:

$$\log_a x = \frac{\ln x}{\ln a}$$

Examples: Change the base

1. $\log_4 25 = \frac{\ln 25}{\ln 4}$

Properties of Logarithms		
1. Product Rule $\log_a xy = \log_a x + \log_a y$	2. Quotient Rule $\log_a \frac{x}{y} = \log_a x - \log_a y$	3. Power Rule $\log_a x^y = y \log_a x$

Examples:

1. Rewrite the following in terms of $\ln 2\ and \ln 3$
   a. $\ln 6 = \ln 2 + \ln 3$
   b. $\ln \frac{2}{27} = \ln 2 - \ln 27 = \ln 2\ - 3 \ln 3$

2. Expand the following:
    a. $\log_4 5x^3y = \log_4 5 + 3\log_4 x + \log_4 y$
    b. $\ln\frac{\sqrt{3x-5}}{7} = \frac{1}{2}(\ln(3x-5) - \ln 7$

3. Condense the following:
    a. $\frac{1}{2}\log_{10} x + 3\ \log_{10}(x+) = \log_{10}\sqrt{x}\,(x+1)^3$

Solve equations with Logs:

1. Sarah invests \$1000 in an account that earns 5.25% compounded annually. How long will it take the account to reach \$2500?

$$y = Pe^{rt}$$
$$2500 = 1000e^{0.0525t}$$
$$2.5 = e^{0.0525t}$$
$$\ln 2.5 = \ln e^{0.0525t}$$
$$\ln 2.5\ = 0.0525t$$
$$t = \ 3.86\ years$$

2. Use properties of logs to solve for y.

$$\ln y\ = 2t + 4$$

3. Solve for x: $\log_2(\log_2 x) = 2$

4. Solve for x: $(\log_3 x)^2 - \log_3 x^2 = 3$