1.10 Inverse Functions

Definition of Inverse Function: Let f and g be two functions such that

$f(g(x)) = x$ for every x in the domain of g

And

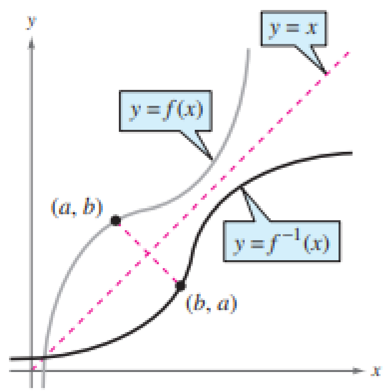
$g(f(x)) = x$ for every x in the domain of f.

Under these conditions, the function g is the **inverse function** of the function f. The function g is denoted by f^{-1} (read f-inverse)

Therefore, $f(f^{-1})) = x$ *and* $f^{-1}(f(x)) = x$.

The domain of f must be equal to the range of f^{-1}, and the range of f must be equal to the domain of f^{-1}.

Graphically, The inverse functions reflect across the line y = x.



One-to-One Function: A function f is one-to-one, if for a and b in its domain, $f(a) = f(b)$ implies that a = b.

Existence of an Inverse Function: A function f has an inverse function f^{-1} if and only if f is one-to-one.

Finding an Inverse Function.

1. Use the Horizontal Line Test to decide whether f has an inverse function.
2. In the equation for f(x), replace f(x) with y.
3. Interchange the roles of x and y, and solve for y.
4. Replace y by $f^{-1}(x)$ in the new equation.
5. Verify that f *and* f^{-1} are inverse functions of each other by showing that the domain of f is equal to the range of f^{-1}, the range of f is equal to the domain of f^{-1}, and $f(f^{-1}(x)) = x$ *and* $f^{-1}(f(x)) = x$.

Example: Find the inverse function of $f(x) = \frac{5-3x}{2}$

$f(x) = \dfrac{5-3x}{2}$	Write the original function
$y = \dfrac{5-3x}{2}$	Replace f(x) with y
$x = \dfrac{5-3y}{2}$	Interchange x and y
$2x = 5 - 3y$	Solve for y.
$2x - 5 = -3y$	
$y = \dfrac{2x-5}{-3}$	
$f^{-1}(x) = \dfrac{2x-5}{-3}$	Replace y with $f^{-1}(x)$

You Try: Find the inverse of each of the functions.

1. $f(x) = x^3$ $f^{-1}(x) = \sqrt[3]{x}$	2. $f(x) = \frac{2x+4}{x-1}$ $f^{-1}(x) = \dfrac{-x-4}{2-x}$